Hawking radiation from black holes in de Sitter spaces via covariant anomalies

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Abstract

We apply the covariant anomaly cancellation method to compute the Hawking fluxes from the event and cosmic horizons of the Schwarzschild-de Sitter black hole. The derivation is new from the existing ones as we split the space in three different regions (near to and away from the event and cosmic horizons) and write down the covariant energy-momentum tensor using three step functions which covers the whole region leading elegantly to the conditions required to compute the Hawking fluxes from the event and cosmic horizons.

Keywords: Hawking radiation, anomaly

Introduction :
In the early seventies, Hawking [1], [2] proposed that black holes evaporate due to quantum particle creation and behave like thermal bodies with an appropriate temperature. This is essentially a consequence of quantisation of matter in a background spacetime having an event horizon. There are several derivations of this effect [3], [4], [5], [6], [7], [8].
Recently, Robinson, Wilczek and collaborators gave an interesting method to compute the Hawking fluxes using chiral gauge and gravitational anomalies [9]. The method was soon extended to the case of charged blackholes [10]. It rests on the fact that the effective theory near the event horizon is a two dimensional chiral theory which, therefore, has gauge and gravitational anomalies. However, an unpleasant feature of [9], [10] was that whereas the expressions for chiral anomalies were taken to be consistent, the boundary condition necessary to fix the parameters were vanishing of covariant current and energy-momentum tensor at the event horizon. A more conceptually cleaner and economical approach based on cancellation of covariant (gauge/gravitational) anomaly has been discussed in [11]. Since the boundary condition involved the vanishing of covariant current/energy-momentum tensor at the horizon, it is more natural to make use of covariant expressions for gauge and gravitational anomaly. The generalization of this approach to non-Schwarzschild black holes has been done in [12, 13].
In this paper, we adopt the method in ([9]) to discuss Hawking radiation from Schwarzschild-de Sitter blackhole. In [14], [15] this was done using consistent anomalies. The novelty in this derivation apart from using covariant expressions for the energy-momentum tensor throughout the analysis is that we split the space in three different regions (near to and away from the event and cosmic horizons) and write down the covariant energy-momentum tensor using three step functions which covers the whole region. This splitting using three step functions is different from the earlier approaches [14], [15]. The method elegantly lead to the conditions required to compute the Hawking fluxes from the event and cosmic horizons.

Hawking radiation from Schwarzschild-de Sitter blackhole :

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The Schwarzschild solution with a positive cosmological constant Λ represents a black hole in asymptotically de Sitter space. The metric of the four dimensional Schwarzschild-de Sitter black hole reads

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega \tag{1}$$

where,

$$f(r) \quad = \quad 1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2 \quad ; \quad \Lambda > 0 \ . \tag{2}$$

It is easy to see that $f(r) = 0$ at two positive values of r when $9\Lambda M^2 < 1$. The smaller root r_H denotes the position of the event horizon and the larger one r_C denotes the position of the cosmic horizon. With the aid of dimensional reduction near the event and cosmic horizons, one can effectively describe a theory with a metric given by the "$r - t$" sector of the full spacetime metric (1) near the two horizons.

Now we divide the spacetime into three regions. In the region outside the event and cosmic horizons ($r_H + \epsilon \leq r \leq r_C - \epsilon$), the theory is free from anomaly and hence we have the energy-momentum tensor satisfying the conservation law

$$\nabla_\mu T^\mu_{(o)\nu} = 0 \ . \tag{3}$$

However, the omission of the ingoing modes in the region $r_H \leq r \leq r_H + \epsilon$ near the event horizon and the omission of the outgoing modes in the region $r_C - \epsilon \leq r \leq r_C$ near the cosmic horizon leads to an anomaly in the energy-momentum tensor in these regions. As we have mentioned earlier, in this paper we shall focus only on the covariant form of $d = 2$ gravitational anomaly given by ([9, 10]):

$$\begin{aligned} \nabla_\mu T^\mu_{(H)\nu} &= \frac{1}{96\pi}\epsilon_{\nu\mu}\partial^\mu R = \mathcal{A}_\nu \\ \nabla_\mu T^\mu_{(C)\nu} &= -\frac{1}{96\pi}\epsilon_{\nu\mu}\partial^\mu R = -\mathcal{A}_\nu \end{aligned} \tag{4}$$

where, $\epsilon^{\mu\nu}$ and $\epsilon_{\mu\nu}$ are two dimensional antisymmetric tensors for the upper and lower cases with $\epsilon^{tr} = \epsilon_{rt} = 1$. It is easy to check that for the metric (1), the anomaly is purely timelike with

$$\begin{aligned} \mathcal{A}_r &= 0 \\ \mathcal{A}_t &= \partial_r N^r_t \end{aligned} \tag{5}$$

where,

$$N^r_t = \frac{1}{96\pi}\left(ff'' - \frac{f'^2}{2}\right) . \tag{6}$$

Now in the region free from anomaly, the conservation equation (3) yields the differential equation

$$\partial_r T^r_{(o)t} = 0 \tag{7}$$

which after integration leads to

$$T^r_{(o)t}(r) = a_o \tag{8}$$

where, a_o is an integration constant. In the region near the event and cosmic horizons, the anomaly equations (4) lead to the following pair of differential equations

$$\begin{aligned} \partial_r T^r_{(H)t} &= \partial_r N^r_t(r) \\ \partial_r T^r_{(C)t} &= -\partial_r N^r_t(r) \end{aligned} \tag{9}$$

which after solution lead to

$$\begin{aligned} T^r_{(H)t}(r) &= b_H + N^r_t(r) - N^r_t(r_H) \\ T^r_{(C)t}(r) &= c_H - N^r_t(r) + N^r_t(r_H) \end{aligned} \tag{10}$$

where, b_H and c_H are integration constants.

Now as in ([10], [11]), writing the energy-momentum tensor as a sum of three contributions

$$T^r{}_t(r) = T^r_{(H)t}(r)H(r) + T^r_{(o)t}(r)[\theta(r - r_H - \epsilon) - \theta(r - r_C + \epsilon)] + T^r_{(C)t}(r)\theta(r - r_C + \epsilon) \tag{11}$$

where, $H(r) = 1 - \theta(r - r_H - \epsilon)$, we find

$$\begin{aligned}
\nabla_\mu T^\mu{}_t &= \partial_r T^r{}_t(r) \\
&= \left(T^r_{(o)t}(r) - T^r_{(H)t}(r) + N^r_t(r)\right)\delta(r - r_H - \epsilon) + \left(T^r_{(C)t}(r) - T^r_{(o)t}(r) + N^r_t(r)\right)\delta(r - r_C + \epsilon) \\
&\quad + \partial_r\left[N^r_t(r)H(r)\right] - \partial_r\left[N^r_t(r)\theta(r - r_C + \epsilon)\right] . \qquad (12)
\end{aligned}$$

The terms involving the total derivatives are cancelled by quantum effects of classically irrelevant ingoing and outgoing modes at the event and cosmic horizon respectively. The quantum effect to cancel these terms is the Wess-Zumino term induced by the ingoing and outgoing modes near the event and cosmic horizons. Now since the full theory must be invariant under diffeomorphism symmetry, hence the energy-momentum tensor must be covariantly conserved, i.e. $\nabla_\mu T^\mu{}_t = 0$. This leads to two separate conditions. When $r = r_H + \epsilon$, $\delta(r - r_C + \epsilon)$ vanishes and hence the coefficient of $\delta(r - r_H - \epsilon)$ must vanish at $r = r_H + \epsilon$ for $\nabla_\mu T^\mu{}_t$ to vanish leading to

$$T^r_{(o)t}(r_H + \epsilon) - T^r_{(H)t}(r_H + \epsilon) + N^r_t(r_H + \epsilon) = 0 \ . \qquad (13)$$

Similarly, when $r = r_C - \epsilon$, $\delta(r - r_H - \epsilon)$ vanishes and hence the coefficient of $\delta(r - r_C + \epsilon)$ must vanish at $r = r_C - \epsilon$ for $\nabla_\mu T^\mu{}_t$ to vanish leading to

$$T^r_{(C)t}(r_C - \epsilon) - T^r_{(o)t}(r_C - \epsilon) + N^r_t(r_C - \epsilon) = 0 \ . \qquad (14)$$

Now to compute the Hawking flux from the event horizon, we focus our attention on (13) and also set the covariant boundary condition $T^r_{(H)t}(r_H) = 0$ which yields $b_H = 0$. Hence, eq.(13) becomes

$$T^r_{(o)t}(r_H + \epsilon) = -N^r_t(r_H) = \frac{1}{192\pi} f'^2(r_H) \ . \qquad (15)$$

Now since $T^r_{(o)t}(r) = T^r_{(o)t}(r_H + \epsilon) = a_o$, therefore the Hawking flux from the event horizon is given by

$$a_o = \frac{1}{192\pi} f'^2(r_H) \ . \qquad (16)$$

To compute the Hawking flux from the cosmic horizon, we focus our attention on (14) and also set the covariant boundary condition $T^r_{(C)t}(r_C) = 0$ which yields $c_H = 0$. Hence, eq.(14) becomes

$$T^r_{(o)t}(r_C - \epsilon) = N^r_t(r_C) = -\frac{1}{192\pi} f'^2(r_C) \ . \qquad (17)$$

Now since $T^r_{(o)t}(r) = T^r_{(o)t}(r_C - \epsilon) = a_o$, therefore the Hawking flux from the cosmic horizon is given by

$$a_o = -\frac{1}{192\pi} f'^2(r_C) \ . \qquad (18)$$

Discussions :

In this paper, we have computed the Hawking flux from Schwarzschild-de Sitter blackhole which has two horizons (event and cosmic) due to the presence of a positive cosmological constant. Unlike the approach in [14], we split the space in a different way so that the energy-momentum tensor can be written down in terms of the energy-momentum tensor near the event horizon (having a chiral anomaly), in the region away from the event and cosmic horizons (anomaly free region) and near the cosmic horizon (having a chiral anomaly) using three step functions. Further, covariant expressions for the energy-momentum tensor have been used throughout the paper.

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